

Multisoliton Newton's cradles and supersolitons in regular and \mathcal{PT} -symmetric nonlinear couplers

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We demonstrate the existence of stable collective excitation in the form of “supersolitons” propagating through chains of solitons with alternating signs (i.e., Newton's cradles built of solitons) in nonlinear optical couplers, including the \mathcal{PT} -symmetric version thereof. In the regular coupler, stable supersolitons are created in the cradles composed of both symmetric solitons and asymmetric ones with alternating polarities. Collisions between moving supersolitons are investigated too, by the means of direct simulations in both the regular and \mathcal{PT} -symmetric couplers.

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I. INTRODUCTION AND THE MODEL

Solitons offer a straightforward realization of the wave-particle dualism in classical settings. Indeed, being built as self-trapped wave pulses, they often behave as particles [1]. Well-known applications of optical solitons to data transmission and (potentially) all-optical data processing make it necessary to design various handling devices, one of basic types of which is the nonlinear directional coupler. Nonlinear twin-core fiber couplers have been a subject of intensive studies since they were proposed in Refs. [2, 3]. Soliton dynamics in them has been analyzed in detail, especially as concern the use of the couplers for the switching of narrow pulses, both theoretically [4–10] and experimentally [11, 12]. Stationary modes supported by the couplers have been studied too, revealing the existence of symmetric [13, 14], antisymmetric, and asymmetric solitons [7, 15–17]. Collisions between solitons of different types were investigated in Ref. [18]. In addition to the symmetric twin-core couplers, static and dynamical modes and their stability were explored in asymmetric couplers [19, 20], including those with opposite signs of the group-velocity dispersion [21, 22].

The analysis of the stability and interactions of solitons has been extended to active dual-core systems, such as the model with gain and loss acting in two coupled cores [25, 26, 29]. Recently, a \mathcal{PT} -symmetric extension of the concept of solitons in nonlinear couplers was introduced, assuming, as it should be in diverse realizations of such systems [30–48], the balance between the gain and loss in the two cores of the coupler [49–53] (in the above-mentioned earlier studied models, the loss acting in the passive core was stronger than the gain in the active one, to secure the stability margin for the resulting dissipative solitons [25, 26, 29]).

The quasi-particle properties of solitons suggest to employ them for emulating various effects from classical me-

chanics in optical settings, with possible applications to the design of compact nonlinear-optical circuitry. One recently elaborated example is a possibility to use arrays of optical solitons, in dissipative two-dimensional [54] and conservative one-dimensional [55] setups alike, for building optical counterparts of the Newton's cradle (NC), which are well known in mechanics and molecular dynamics [56–59], and “supersolitons”, i.e., self-supporting dislocations propagating in chains of individual solitons. Previously, “supersolitons” were experimentally realized and theoretically studied in chains of fluxons populating long periodically inhomogeneous Josephson junctions [60, 61], and predicted in binary Bose-Einstein condensates (BECs), with attractive interactions in each component and repulsion between them [62]. In this work, we aim to develop a robust NC model, using soliton chains in nonlinear couplers, and construct stable supersolitons as localized collective excitations in such chains.

The propagation of optical pulses in a nonlinear dual-core fiber coupler can be described by two linearly coupled nonlinear Schrödinger (NLS) equations. This model was applied to the soliton switching in erbium-doped nonlinear fiber couplers [23] and passively mode-locked lasers [24], and to the prediction of stable solitons in two-component active systems [25]. In this paper, we start with the coupled NLS equations similar to those adopted in Ref. [26]:

$$iU_Z + (1/2)U_{TT} + |U|^2 U + KV = i\Gamma_1 U, \quad (1)$$

$$iV_Z + (1/2)V_{TT} + |V|^2 V + KU = -i\Gamma_2 V, \quad (2)$$

where $U(Z, T) = NA_U(z, t)/\sqrt{P_0}$ and $V(Z, T) = NA_V(z, t)/\sqrt{P_0}$ are the normalized modal field amplitudes in the two cores, $Z = z/L_D$ and $T = (t - z/v_g)/T_0$ are the normalized length and time in soliton units, $A_{U,V}(z, t)$ are slowly varying amplitudes of the electromagnetic waves [27], $K = L_D C$ is proportional to the linear coupling coefficient C of the dual-core fiber, while $\Gamma_1 = gL_D/2$ and $\Gamma_2 = \alpha L_D/2$ represent the normalized linear gain and loss in the active and passive cores. Further, T_0 is the width of the incident pulse, v_g is the group

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velocity, L_D , L_{NL} and N are, respectively, the dispersion length, nonlinearity length, and soliton order, defined as

$$L_D = T_0^2 / |\beta_2|, L_{NL} = 1 / (\gamma P_0), N = \sqrt{L_D / L_{NL}}, \quad (3)$$

where $\beta_2 < 0$ and γ are the group-velocity dispersion (GVD) and effective Kerr coefficient of the fiber, and, finally, g and α are the effective gain, provided by dopants in the active core, and loss in the passive one [28]. The same equations with $g = \alpha$, i.e., $\Gamma_1 = \Gamma_2$, represent the model of the parity-time (\mathcal{PT})-symmetric coupler, with mutually balanced gain and loss [49]. The present model neglects higher-order effects, such as the third-order dispersion and the shock and Raman terms.

The usual model of the symmetric coupler implies the that the propagation length is much smaller than the dissipation length, hence $\Gamma_1 = \Gamma_2 = 0$ is assumed, and Eqs. (1) and (2) can be rescaled ($U \equiv \sqrt{K}u$, $V \equiv \sqrt{K}v$, $T \equiv \tau/\sqrt{K}$, $\zeta \equiv KZ$) to a form devoid of any free parameter:

$$iu_\zeta + (1/2)u_{\tau\tau} + |u|^2 u + v = 0, \quad (4)$$

$$iv_\zeta + (1/2)v_{\tau\tau} + |v|^2 v + u = 0. \quad (5)$$

While Eqs. (1) and (2) are written in terms of the temporal-domain propagation, which corresponds to the couplers in the form of twin-core nonlinear optical fibers, the model may be realized in the spatial domain as well, pertaining to a twin-core planar optical waveguide, with temporal variable τ replaced by transverse coordinate x . The same system, with ζ and τ replaced, respectively, by time and x , plays the role of Gross-Pitaevskii equations for the BEC trapped in a dual-core prolate waveguide [63].

It is well known that the system of equations (4), (5) gives rise to symmetric and asymmetric soliton solutions [4–19], whose stability depends on the total energy (norm),

$$Q = \int_{-\infty}^{+\infty} (|u|^2 + |v|^2) d\tau \equiv Q_u + Q_v, \quad (6)$$

or, in other words, on the corresponding propagation constant, p , as shown by means of the $Q(p)$ dependence in Fig. 1 (it actually reproduces Fig. 2 from Ref. [19]). The transition from symmetric to asymmetric solitons, with the increase of Q , occurs via a subcritical symmetry-breaking bifurcation, which features a narrow region of bistability.

The rest of the paper is organized as follows. In Section II, collective excitations in the form of supersolitons in chains of symmetric and asymmetric solitary pulses are studied in the model of the ordinary symmetric coupler. The analysis of the \mathcal{PT} -symmetric generalization of the model is reported in Section III. Conclusions are presented in Section IV.

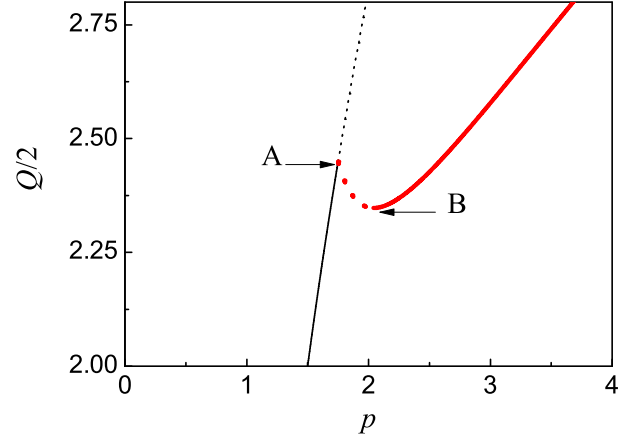


FIG. 1: (Color online) The bifurcation diagram for solitons in the symmetric coupler (without gain and loss), in terms of the dependence between the total energy (Q) and propagation constant (p). The diagram was produced by means of the variational approximation in Ref. [19]. The solid and dot curves correspond to stable and unstable solutions, respectively, while the light black and thick red curves represent, severally, symmetric and asymmetric solitons. Q_s (point A) is the point of the subcritical symmetry-breaking bifurcation, while Q_a (point B) indicates the related tangent bifurcation which gives rise to the pair of stable and unstable asymmetric solitons.

II. SUPERSOLITONS IN SOLITON CHAINS IN THE NONLINEAR COUPLER

Symmetric and antisymmetric solitons are represented by obvious exact solutions to Eqs. (4) and (5), which reduce to the classical NLS (nonlinear-Schrödinger) solitons [7]:

$$u = \pm v = \sqrt{2\beta} \text{sech} \left(\sqrt{2\beta} \tau \right) \exp [i(\beta \pm 1) \zeta], \quad (7)$$

where $\beta \pm 1$ are propagation constants of the symmetric and antisymmetric solutions, respectively. For the symmetric solution, $\beta + 1$ is the propagation constant which is denoted as p in Fig. 1. It is well known that the symmetric solution is stable at

$$\beta \leq \beta_{\max} = 2/3, \quad (8)$$

and unstable at $\beta > \beta_{\max}$ [13]. The total energy of solitons (7) is $Q = 4\sqrt{2\beta}$, hence the corresponding critical value is

$$Q_s^{(\text{exact})} = 8/\sqrt{3} \approx 4.62, \quad (9)$$

Note that Fig. 1 presents a counterpart of this result predicted by the variational approximation, $Q_s^{(\text{var})} = 2\sqrt{6} \approx 4.90$ [19]. Because, as mentioned above, the symmetry-breaking bifurcation is subcritical, pairs of stable and unstable asymmetric solitons appear, via the tangent bifurcation at $Q > Q_a$, see Fig. 1. Unlike the exact

critical point (9), the value of Q_a is known in an approximate form, produced by the variational method: $Q_a^{(\text{var})} = 3 \cdot 6^{1/4} \approx 4.70$ [19], while its numerically generated counterpart can be taken from Fig. 11 of Ref. [17] which is somewhat smaller,

$$Q_a^{(\text{num})} \approx 4.58. \quad (10)$$

A narrow region of the *bistability* of the asymmetric and symmetric solitons is $Q_a^{(\text{num})} < Q < Q_s^{(\text{exact})}$. For the antisymmetric solitons, the effective stability area is much smaller than for the symmetric ones [16], and, strictly speaking, all the antisymmetric solitons are subject to weak instability [52], therefore we do not consider antisymmetric solutions below.

Using stable individual symmetric solitons, whose energy does not exceed the threshold value (9), one can construct a soliton chain with alternating signs of adjacent solitons:

$$(u, v), (-u, -v), (u, v), \dots, (-1)^{n-1} (u, v). \quad (11)$$

The opposite signs are necessary to guarantee repulsion between neighboring solitons, otherwise the chains will be obviously unstable. Experimentally, the alternating signs of temporal solitons in the dual-core fiber can be provided by a modulator combined with the laser source generating the pulses.

Using these chains, one can initiate the NC dynamics by *kicking* one soliton, i.e., multiplying its both components by $\exp(ik\tau)$ with kick strength k . The Galilean invariance of Eqs. (4) and (5) implies that $\text{sech}(\sqrt{2\beta}\tau)$ in solution (7) is then replaced by $\text{sech}[\sqrt{2\beta}(\tau - k\zeta)]$, i.e., the kicked soliton is *boosted* at rate k . Thus the configuration represented by Eq. (11) gives rise to the NC built as the chains of optical solitons, where the kicked one plays the role of the impacting ball in the mechanical realization of the NC.

The soliton chain with free edges will spontaneously expand because of the repulsion between solitons, therefore the simulation were run in the system with periodic boundary conditions, which may physically correspond to optical solitons in a fiber loop [64, 65], or to the BEC in a toroidal trap [66–68]. Figure 2 presents results of simulations of Eqs. (4) and (5) for the evolution of the chain of symmetric solitons, excited by boosting the third soliton. The figure clearly demonstrates that the momentum, which was originally imparted to the third soliton, is transferred along the chain by particle-like elastic collisions between the solitons. The collective excitation mode propagating along the soliton chains may be identified as a “supersoliton”, i.e., a self-supporting localized perturbation moving along the chains of “primary” solitons [62].

Collisions between two supersolitons are further illustrated in the Figs. 3(a) and (b), which show elastic overtaking and head-on collisions in the chain built of symmetric solitons. Multi-supersoliton collisions may be also initiated by kicking all solitons in the chains, as shown in

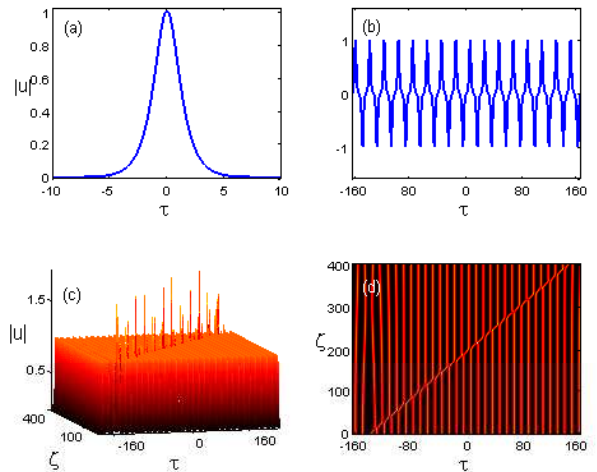


FIG. 2: (Color online) The dynamics of the Newton’s cradle in the chain of identical symmetric solitons hosted by the coupler. (a) An individual symmetric solution with total energy $Q = 4$. (b) The chain built of such solitons with initial separation between adjacent solitons $\Delta\tau = 10$ and $n = 32$. (c) The evolution of the chain excited by boosting the third soliton with strength $k_3 = 0.5$. (d) The top view corresponding to (c). In all panels, only the u component of the mutually symmetric fields (u, v) is displayed.

Figs. 3(c) and (d). The character of the excitation of the chain in the configuration displayed in Fig. 3(c) makes the number of collisions for each soliton a function of its original position, the solitons located closer to central position colliding a larger number of times. The simulations performed with a larger value of the kick strength demonstrate that the left and right parts of the chain perform collectively recurrent elastic collisions, as shown in Fig. 3(d).

As said above, the symmetric solitons are unstable at $Q > Q_s$ [see Eq. (9)], therefore in this case it is relevant to build chains of asymmetric solitons. Because exact solutions for asymmetric modes are not available, they can be created, using the initial guess provided by the variational approximation, which was developed in Ref. [19] on the basis of a simple *ansatz*,

$$u = A \cos(\theta) \text{sech}(\tau/a), v = A \sin(\theta) \text{sech}(\tau/a), \quad (12)$$

with amplitude A , width a , and angle θ which determines the asymmetry of the energy splitting between the cores, $Q_v/Q_u = \tan^2\theta$. Starting from this *ansatz* (with parameters A , a and θ taken from Ref. [19]), numerically exact asymmetric solutions were constructed by means of the Newton’s method, as shown in Fig. 4(a,b). In panel (a) of the figure, ratios of the amplitudes and energies in the two components of the asymmetric soliton are $A_u/A_v = 3.48$ and $Q_v/Q_u = 0.116$, respectively. The total energy of the asymmetric soliton, $Q = 5$, exceeds the minimum value (10), therefore it is stable. As mirror im-

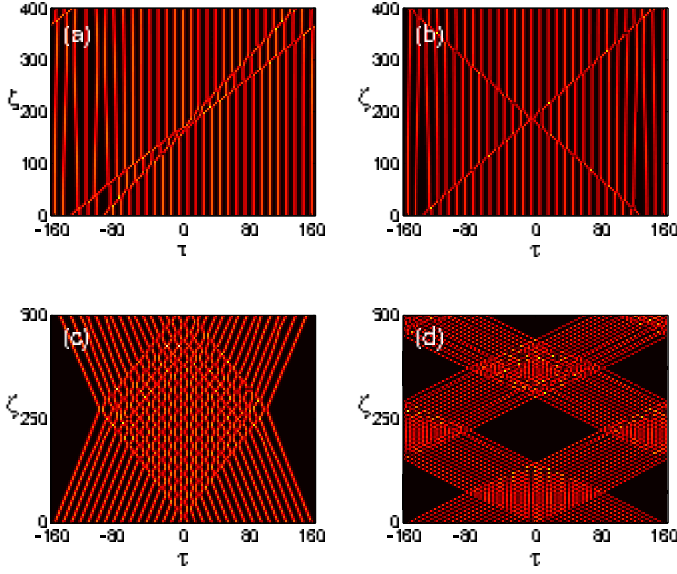


FIG. 3: (Color online) Collisions between supersolitons created in the same chain as in Fig. 2. (a) An overtaking collision of supersolitons excited by kicks with $k_3 = 0.6$ and $k_7 = 0.4$. (b) A head-on collision excited by $k_3 = -k_{29} = 0.5$. (c,d) Cascades of multiple collisions when the kick is applied as follows: (c) $k_1 = \dots = k_{16} = 0.2$ and $k_{17} = \dots = k_{32} = -0.2$; (d) $k_1 = \dots = k_{16} = 1$ and $k_{17} = \dots = k_{32} = -1$. Other parameters are the same as in Fig. 2.

ages of stable asymmetric solitons with $A_u/A_v > 1$, there exist solitons with the opposite *polarity*, i.e., $A_u/A_v < 1$.

Thus, a chain of asymmetric solitons with alternating polarities and phase shifts π between adjacent ones [cf. Eq. (11)] can be build in the form of

$$(u, v), (-v, -u), (u, v), (-v, -u), \dots, \quad (13)$$

as shown in Fig. 4(b). Then, similar to the case of the chain composed of symmetric solitons, the NC dynamics can be initiated in this chain by kicking both components of a selected soliton see Fig. 4(c). A chain can also be composed of asymmetric solitons with identical polarities, but that case seems less interesting. In this context, it is relevant to mention that collision between a stable asymmetric soliton and its counterpart with the opposite polarity was first studied, by means of direct simulations, in Ref. [18].

Figure. 4(c) demonstrate that collective supersolitons can be created in the NC of the present type, if the strength of the initial kick is selected appropriately. An elastic head-on collisions between two supersolitons is displayed in Fig. 4(d).

The NC and supersolitons propagating in it can be built too in a chain of strongly asymmetric solitons, as shown in Fig. 5, where the total energy of the individual soliton is $Q = 7.74$, with the ratios of the amplitudes and energies of the two components being $A_u/A_v = 10.5$ and $Q_v/Q_u = 0.0134$, see Fig. 5(a). For an appropriately chosen initial kick, the NC dynamics in the present chain

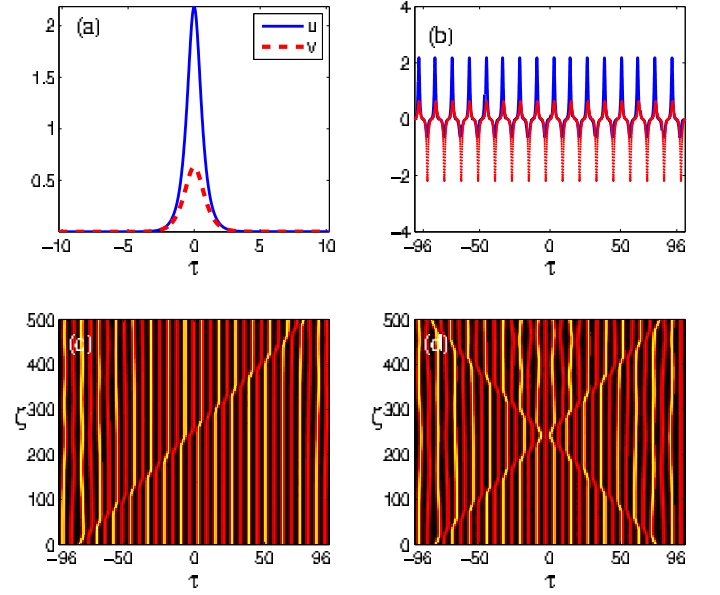


FIG. 4: (Color online) (a) Profiles of the two field components u (the solid blue line) and v (dashed red line) of the stable asymmetric soliton with total energy $Q = 5$. (b) Chains of asymmetric solitons with alternating polarities and opposite signs of adjacent solitons in the u -core (solid blue line) and v -core (dot red line), with the initial separation between adjacent solitons $\Delta\tau = 6$ and $n = 32$. (c) The propagation of a supersoliton, initiated by the application of the kick with $k_3 = 0.15$ to the third soliton in array (11). Here components $|u|$ and $|v|$ are shown by yellow and red, respectively. (d) The head-on collision between supersolitons created in the same chain as in (c), by the application of kicks $k_3 = -k_{29} = 0.15$.

is similar to that observed in Fig. 4. In particular, the collision between supersolitons, which is shown in 5(d), is again elastic.

The size of the initial kick is important for the excitation of supersolitons in the chains of asymmetric solitons with alternating polarities. Figures 6 and 7 display the evolution of the chain with energies of individual solitons $Q = 5$ and $Q = 7.74$, for different kicks. It is observed that the chains are destabilized by increasing the strength of the kick. This is different from the chains of symmetric solitons, which remain robust even under the action of strong kicks.

III. THE EXCITATION OF SUPERSOLITON IN \mathcal{PT} -SYMMETRIC COUPLER

As shown in detail in Refs. [49–53], a \mathcal{PT} -symmetric coupler can be constructed, on the basis of the usual one considered above, by adding linear-gain and loss terms with equal coefficients (Γ_0) to Eqs. (4) and (5), respectively [which corresponds to $\Gamma_1 = \Gamma_2 = \Gamma_0$ in Eqs. (1)

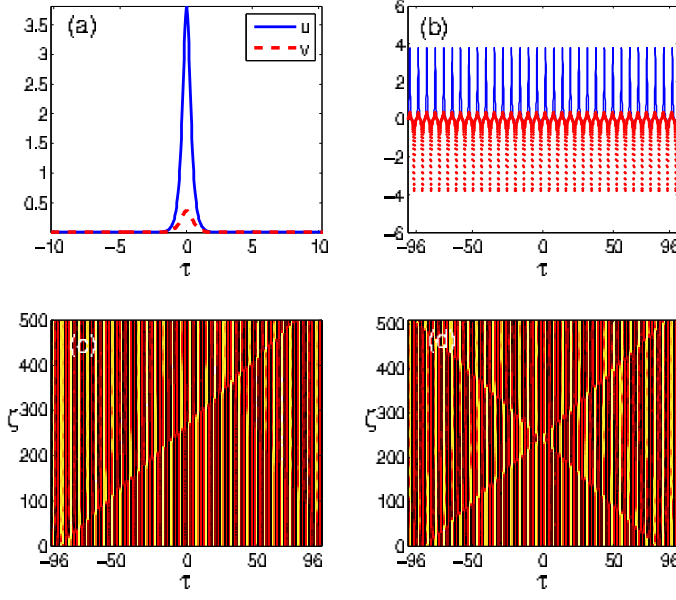


FIG. 5: (Color online) The same as in Fig. 4, but for a chain built of strongly asymmetric solitons, each with total energy $Q = 7.74$. The separation between adjacent solitons and their number are $\Delta\tau = 3$ and $n = 64$. The initial kicks are $k_3 = 0.1$ in (c), and $k_5 = -k_{59} = 0.1$ in (d). Other parameters are the same as in Fig. 4.

and (2)]:

$$iu_\zeta + (1/2)u_{\tau\tau} + |u|^2 u + v = i\Gamma_0 u, \quad (14)$$

$$iv_\zeta + (1/2)v_{\tau\tau} + |v|^2 v + u = -i\Gamma_0 v. \quad (15)$$

Further, the substitution of

$$\Phi(\tau, \zeta) \equiv v(\tau, \zeta) = \left(i\Gamma_0 \pm \sqrt{1 - \Gamma_0^2} \right) u(\tau, \zeta), \quad (16)$$

transforms Eqs. (4) and (5) into a single equation for Φ :

$$i\Phi_\zeta + (1/2)\Phi_{\tau\tau} + |\Phi|^2 \Phi \pm \sqrt{1 - \Gamma_0^2} \Phi = 0, \quad (17)$$

provided that $\Gamma_0 \leq 1$, hence the \mathcal{PT} -symmetric and antisymmetric solitons, which correspond, respectively, to the upper and lower signs \pm in Eqs. (4) and (5), are found as

$$\Phi = \sqrt{2\beta} \text{sech} \left(\sqrt{2\beta} \tau \right) \exp \left[i \left(\beta \pm \sqrt{1 - \Gamma_0^2} \right) \zeta \right]. \quad (18)$$

For the \mathcal{PT} -symmetric solitons, the boundary of the stability region is

$$\beta \leq \beta_{\max} = 2\sqrt{1 - \Gamma_0^2}/3 \quad (19)$$

[49, 52], while asymmetric solitons do not exist in this system. \mathcal{PT} -antisymmetric solitons [recall they correspond to the lower sign \pm in Eq. (16)] are, strictly speaking,

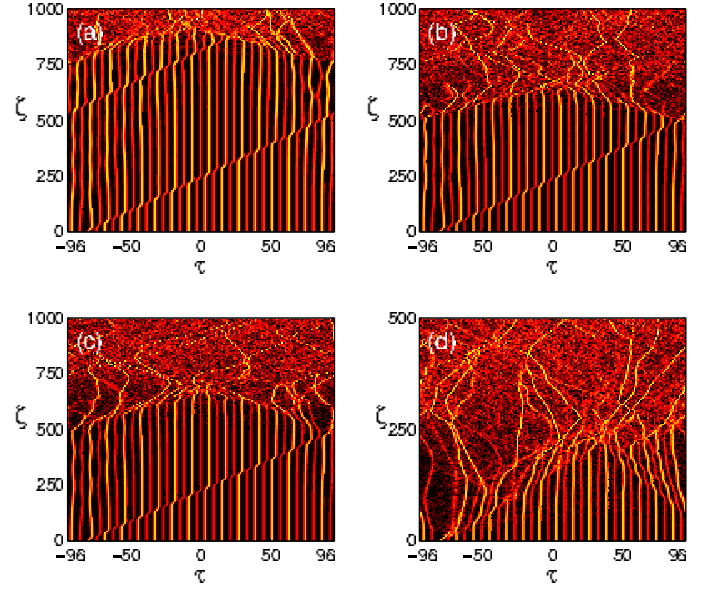


FIG. 6: (Color online) The evolutions of the chain built of asymmetric solitons with the individual energy $Q = 5$, for the different strengths of the initial kick: (a) $k_3 = 0.16$, (b) $k_3 = 0.17$, (c) $k_3 = 0.18$ and (d) $k_3 = 0.5$. The other parameters are the same as in Fig. 4.

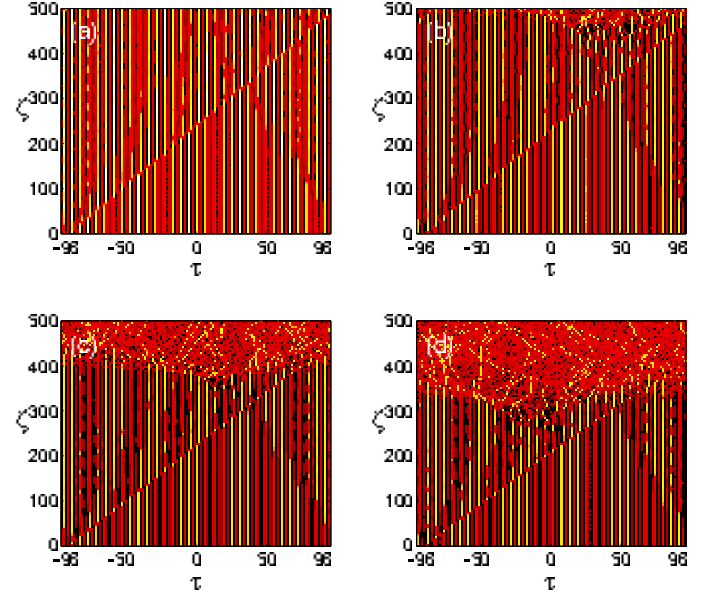


FIG. 7: (Color online) The same as in Fig. 6, but for $Q = 7.74$: (a) $k_3 = 0.13$, (b) $k_3 = 0.14$, (c) $k_3 = 0.15$ and (d) $k_3 = 0.17$. The other parameters are the same as in Fig. 5.

always unstable [52], but, in some domain [which is essentially smaller than the stability area (19) of the symmetric solitons], the antisymmetric solitons seem practically stable in direct simulations, as the underlying instability is weak [49].

One can construct a \mathcal{PT} -symmetric soliton chains, and launch the NC dynamics in it, following the same pat-

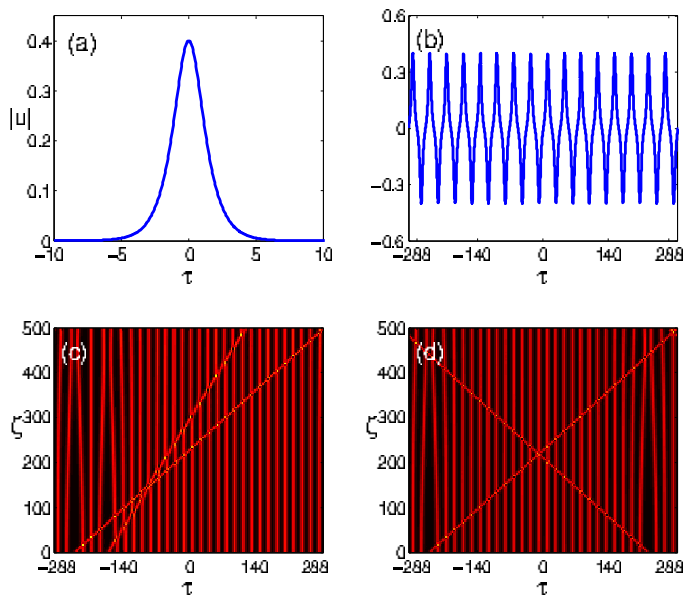


FIG. 8: (Color online) The supersoliton in the chain of symmetric solitons housed by the \mathcal{PT} -symmetric coupler with the gain-loss coefficient $\Gamma_0 = 0.5$. (a) The profile of an individual stable symmetric soliton with total energy $Q = 1.6$. (b) A chain composed of 32 such solitons with initial separation between adjacent solitons $\Delta\tau = 18$. (c) An elastic overtaking collision between the supersolitons excited by kicks $k_3 = 1$ and $k_7 = 0.5$. (d) A head-on collision of the supersolitons excited by $k_3 = -k_{29} = 1$.

tern as elaborated above. The corresponding numerical results are displayed in Fig. 8 (note that the possibility of elastic collisions between individual stable \mathcal{PT} -symmetric solitons was demonstrated in Ref. [49]).

Figure 8 demonstrates that the propagation of supersolitons in the \mathcal{PT} -symmetric coupler is qualitatively similar to that in the usual nonlinear coupler, because collisions between individual solitons are elastic. A difference from the soliton NC in the ordinary coupler is that the application of the kick to a particular soliton creates a conspicuous perturbation of the NC around that site, which then gradually relaxes. The simulations also reveal the possibility of the propagation of supersolitons in a chain composed of \mathcal{PT} -antisymmetric solitons (not shown here), a natural difference being that the corresponding stability domain is smaller than that in the chain of symmetric solitons.

IV. CONCLUSIONS AND DISCUSSIONS

In this work, we have demonstrated that the models of the usual and \mathcal{PT} -symmetric nonlinear couplers admit the realization of the NC (Newton's cradle), in the form of a stable chain of optical solitons with alternat-

ing signs, and the creation of stable “supersolitons” in the NC, in the form of self-supporting localized collective excitations in the chains, propagating through consecutive elastic collisions between individual solitons. In the usual coupler, these results were demonstrated for the chains composed of both symmetric solitons and asymmetric ones, with alternating polarities. The elastic collisions between supersolitons were also found in usual and \mathcal{PT} -symmetric nonlinear couplers.

To estimate the predicted effects in physical units, we use real fiber parameters to evaluate the corresponding propagation distances, pulse widths, amplitudes, separations between adjacent solitons, the coupling coefficient in the usual nonlinear couplers, built of cores with strong GVD and Kerr nonlinearity (otherwise, the necessary propagation distance will be too large), and the gain/loss constant in the \mathcal{PT} -symmetric counterpart. For this purpose, the GVD and Kerr coefficients of the fiber coupler are taken as $\beta_2 = -9.95 \times 10^2 \text{ ps}^2/\text{km}$ and $\gamma = 10\text{W}^{-1}\text{km}^{-1}$ (such large values are available, e.g., in microstructured fibers [69]), while the soliton order and dimensionless coupling constant are set to be $N = K = 1$. Thus, for a given initial power P_0 , which should be high enough to secure a relatively short nonlinearity distance (for example, $P_0 = 10 \text{ W}$), the propagation-distance and temporal scales, L_{NL} , L_D and T_0 , can be identified [see Eq. (3)], and then the pulse amplitudes (A_U , A_V), widths (W_U , W_V), propagation distance z , coupling coefficient C , and gain g , along with loss α , may be retrieved in physical unit. In this case, normalized propagation distance $\zeta = 500$ in the figures corresponds to the fiber length $\sim 5 \text{ km}$, with the coupling coefficient $\sim 0.1 \text{ m}^{-1}$. The the corresponding pulse widths (W_U , W_V) in Figs. 2 and 3, Figs. 4 and 6, Figs. 5 and 7, and Fig. 8 are, respectively, (2.86, 2.86) ps, (1.35, 1.83) ps, (0.75, 1.10) ps, and (7.1, 7.1) ps, while the temporal separations between adjacent solitons are 31.5 ps, 19.0 ps, 9.5 ps, and 56.8 ps. For the \mathcal{PT} -symmetric coupler, the gain and loss coefficients are 0.43 dB/km. The propagation distance can be further reduced by increasing the input power or using highly-nonlinear fibers [70].

A challenging possibility is to generalize the analysis for chains and lattices of regular [71] and \mathcal{PT} -symmetric [72] spatiotemporal solitons in two-dimensional couplers with the cubic-quintic nonlinearity, which support individual stable solitons of such types.

V. ACKNOWLEDGEMENT

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